Hoofdstuk 15: Continue dynamische modellen

§15.1 Dynamische modellen

Opgave 1:

a. op $[10,20]$: $\frac{\Delta T}{\Delta t} = \frac{34-50}{20-10} = -1{,}6$ ºC/min

op $[20,30]$: $\frac{\Delta T}{\Delta t} = \frac{26-34}{30-20} = -0{,}8$ ºC/min

b. op $[0,10]$: $\frac{\Delta T}{\Delta t} = -4$

dus $\left[\frac{dT}{dt}\right]_{t=5} = -4$

op $t = 5$ geldt: $T = \frac{90+50}{2} = 70$

dus $\frac{dT}{dt} = c(T-20)$ wordt

$-4 = c(70-20)$

$c = -0{,}08$

c. $[10,20]$: $\frac{\Delta T}{\Delta t} = -1{,}6$

$t = 15$ dan $T = \frac{50+34}{2} = 42$

$-1{,}6 = c(42-20)$

$c = -0{,}073$

$[20,30]$: $\frac{\Delta T}{\Delta t} = -0{,}8$

$t = 25$ dan $T = \frac{34+26}{2} = 30$

$-0{,}8 = c(30-20)$

$c = -0{,}08$

d. $c = \frac{-0{,}08+-0{,}073+-0{,}08}{3} = -0{,}078$

$\frac{dT}{dt} = -0{,}078(T-20)$

$\frac{dT}{dt} = -0{,}078T + 1{,}55$

Opgave 2:

a. als je de tijdseenheid verandert, dan krijg je een ander model

b. op $t = \frac{1}{12}$ is $T = 197$ en $\frac{dT}{dt} = \frac{194-200}{\frac{1}{6}} = -36$

$-36 = c(197-15)$

$c = -0{,}198$

$\frac{dT}{dt} = -0{,}198(T-15)$

Opgave 3:

a. $\frac{dT}{dt} = c(T-12)$

$t = 5$ dan $T = \frac{100+94}{2} = 97$ en $\frac{dT}{dt} = \frac{94-100}{10} = -0{,}6$

$-0{,}6 = c(97-12)$

$c = -0{,}007$

$\frac{dT}{dt} = -0{,}007(T-12)$

b. $t = \frac{1}{12}$ dan $T = 97$ en $\frac{dT}{dt} = \frac{94-100}{\frac{1}{6}} = -36$

$-36 = c(97-12)$

$c = -0{,}424$

$\frac{dT}{dr} = -0{,}424(T - 12)$

Opgave 4:

a. $t = 2\frac{1}{2}$ dan $T = \frac{7+9{,}8}{2} = 8{,}4$ en $\frac{dT}{dt} = \frac{9{,}8-7}{5} = 0{,}56$

$0{,}56 = c(8{,}4 - 21)$

$c = -0{,}044$

$\frac{dT}{dt} = -0{,}044(T - 21)$

b. op $[5,10]$:

$t = 7\frac{1}{2}$ dan $T = \frac{12{,}1+9{,}8}{2} = 10{,}95$ en $\frac{dT}{dt} = \frac{12{,}1-9{,}8}{5} = 0{,}46$

$0{,}46 = c(10{,}95 - 21)$

$c = -0{,}046$

op $[20,30]$:

$t = 25$ dan $T = \frac{17{,}4+15{,}3}{2} = 16{,}35$ en $\frac{dT}{dt} = \frac{17{,}4-15{,}3}{10} = 0{,}21$

$0{,}21 = c(16{,}35 - 21)$

$c = -0{,}045$

Opgave 5:

$\frac{dN}{dt} = c \cdot N$

$t = \frac{1}{120}$ dan $N = 10^9 + 2 \cdot 10^6 = 1{,}002 \cdot 10^9$

$\frac{dN}{dt} = \frac{4 \cdot 10^6}{\frac{1}{60}} = 2{,}4 \cdot 10^8$

$2{,}4 \cdot 10^8 = c \cdot 1{,}002 \cdot 10^9$

$c = 0{,}24$

$\frac{dN}{dt} = 0{,}24N$

Opgave 6:

a. $\frac{dv}{dt} = c \cdot v$

$t = 13$ dan $v = \frac{30+24}{2} = 27$ en $\frac{dv}{dt} = \frac{24-30}{6} = -1\ {}^{m}\!/\!_{s^2}$

$-1 = c \cdot 27$

$c = -0{,}037$

$\frac{dv}{dt} = -0{,}037v$

b. $t = \frac{13}{60}$ dan $v = 27\ {}^{m}\!/\!_{s}$ en $\frac{dv}{dt} = \frac{24-30}{\frac{6}{60}} = -60\ {}^{m}\!/\!_{s}$ per minuut

$-60 = c \cdot 27$

$c = -2{,}22$

$\frac{dv}{dt} = -2{,}22v$ met v in ${}^{m}\!/\!_{s}$ en t in minuten

Opgave 7:

op $t = 0$ is $Z = 5000$ g

instroom: 0,5g

als er Z gram zout in het vat zit dan is de uitstroom $0{,}5 \cdot \frac{Z}{1000} = 0{,}0005Z$

dus $\frac{dZ}{dt} = 0{,}5 - 0{,}0005Z$

Opgave 8:

a. $36\,{}^{km}\!/\!_{uur} = 10\,{}^{m}\!/\!_{s}$

$F_W = F_V$ dus $c \cdot 10^2 = 50$

$c = 0{,}5$

b. $24\,{}^{km}\!/\!_{uur} = 6\tfrac{2}{3}\,{}^{m}\!/\!_{s}$

$F_W = 0{,}5 \cdot (6\tfrac{2}{3})^2 = 22{,}2\,\text{N}$

$F_R = F_V - F_W = 50 - 22{,}2 = 27{,}8\,\text{N}$

c. $F = m \cdot a$

$27{,}8 = 100a$

$a = 0{,}278\,{}^{m}\!/\!_{s^2}$

d. $F_R = F_V - F_W = 50 - 0{,}5v^2$

$50 - 0{,}5v^2 = 100 \cdot \frac{dv}{dt}$

$\frac{dv}{dt} = 0{,}5 - 0{,}005v^2$

Opgave 9:

De grenswaarde wordt bereikt als $F_R = 0$

$120 - 10v^2 = 0$

$-10v^2 = -120$

$v^2 = 12$

$v = \sqrt{12} = 2\sqrt{3}$

Opgave 10:

a. $\frac{dy}{dt} = 0$ geeft $5y(10 - y) = 0$

$y = 0 \quad \vee \quad y = 10$

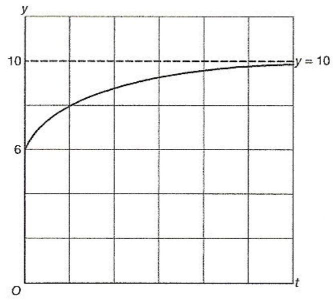
voor $0 < y < 10$ geldt $\frac{dy}{dt} > 0$

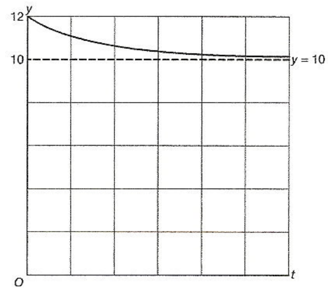
dus de grafiek stijgt

b. $\frac{dy}{dt} < 0$ dus de grafiek daalt

c. als $\frac{dy}{dt} < 0$ dus voor $y(0) < 0 \quad \vee \quad y(0) > 10$

daalt de grafiek





Opgave 11:

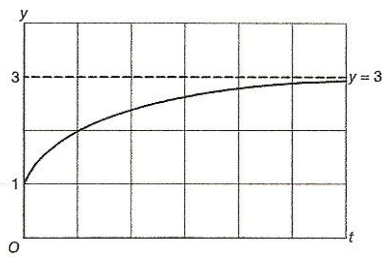
a. $\frac{dy}{dt} = 3y - y^2 = 0$

$y(3 - y) = 0$

$y = 0 \quad \vee \quad y = 3$

voor $0 < y < 3$ is $\frac{dy}{dt} > 0$

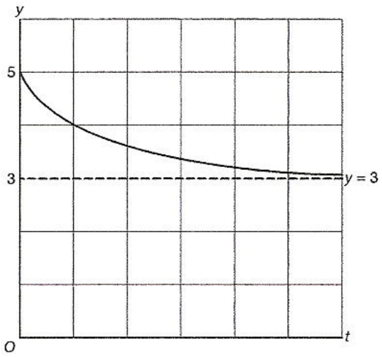
dus de grafiek stijgt



b. voor $y > 3$ is $\frac{dy}{dt} < 0$
dus de grafiek daalt

c. $\frac{dy}{dt} < 0$ dus $y(0) < 0 \quad \vee \quad y(0) > 3$

d. $y(0) = 0 \quad \vee \quad y(0) = 3$



Opgave 12:

a. $F_R = F_Z - F_W = 882 - \frac{1}{4}v^2$

$F = m \cdot a$ dus $882 - \frac{1}{4}v^2 = 90 \cdot \frac{dv}{dt}$

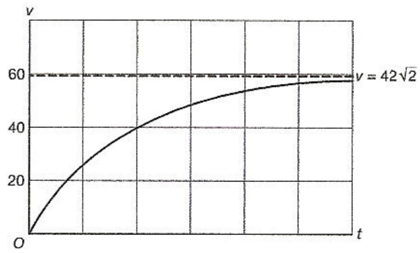
$\frac{dv}{dt} = 9{,}8 - 0{,}0028v^2$

b. $\frac{dv}{dt} = 9{,}8 - 0{,}0028v^2 = 0$

$-0{,}0028v^2 = -9{,}8$

$v^2 = 3528$

$v = \sqrt{3528} = 42\sqrt{2} = 59{,}4$



Opgave 13:

a. $F_R = F_S - F_L - F_W = 50 - \frac{1}{4}v^2 - 4v$

$F = m \cdot a$ geeft $50 - \frac{1}{4}v^2 - 4v = 125 \cdot \frac{dv}{dt}$

$\frac{dv}{dt} = 0{,}4 - 0{,}002v^2 - 0{,}032v$

b. $\frac{dv}{dt} = 0$

$-0{,}002v^2 - 0{,}032v + 0{,}4 = 0$

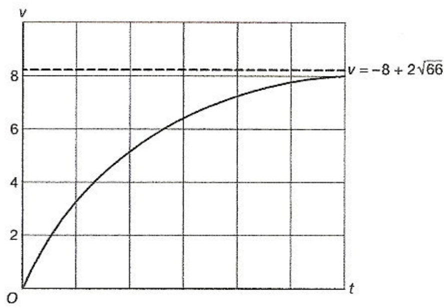
$v^2 + 16v - 200 = 0$

$v = \frac{-16 \pm \sqrt{1056}}{2} = 8{,}25 \, {}^{m/s}\!\!/ = 29{,}7 \, {}^{km}\!/_{uur}$

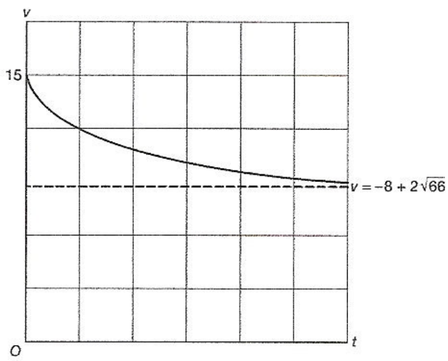
dus de snorfietser kan harder dan 25 ${}^{km}\!/_{uur}$

c. $\frac{dv}{dt} = 0{,}4 - 0{,}002 \cdot 5^2 - 0{,}032 \cdot 5 = 0{,}19 \, {}^{m}\!/_{s^2}$

d. $\frac{dv}{dt} > 0$
dus de grafiek stijgt



e. $\frac{dv}{dt} < 0$ dus de grafiek daalt



Opgave 14:

a. $F_W = c \cdot v^2 = 2670v^2$

$F_R = F_S - F_W = 600000 - 2670v^2$

$F = m \cdot a$ dus $600000 - 2670v^2 = 3500000 \cdot \frac{dv}{dt}$

$\frac{dv}{dt} = \frac{6}{35} - \frac{267}{350000} v^2$

b. $\frac{dv}{dt} = 0$ dus $\frac{6}{35} - \frac{267}{350000} v^2 = 0$

$-\frac{267}{350000} v^2 = -\frac{6}{35}$

$v^2 = 224{,}7$

$v = 15{,}0$

c. $F_R = -F_S - F_W = -600000 - 2670v^2$

$F = m \cdot a$ dus $-600000 - 2670v^2 = 3500000 \cdot \frac{dv}{dt}$

$\frac{dv}{dt} = -\frac{6}{35} - \frac{267}{350000} v^2$

d. $F_R = -F_S + F_W = -600000 + 2 \cdot 2670v^2 = -600000 + 5340v^2$

$F = m \cdot a$ dus $-600000 + 5340v^2 = 3500000 \cdot \frac{dv}{dt}$

$\frac{dv}{dt} = -\frac{6}{35} + \frac{267}{175000} v^2$

e. bij vooruit varen is de formule van de asymptoot $v = 15{,}0$

bij het afremmen neemt de snelheid af tot 0, daarna gaat het schip achteruit varen

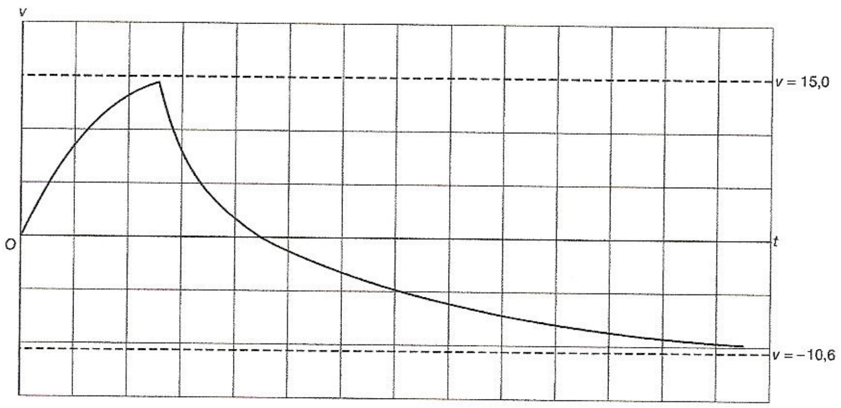
bij het achteruit varen is de asymptoot:

$\frac{dv}{dt} = -\frac{6}{35} + \frac{267}{175000} v^2 = 0$

$\frac{267}{175000} v^2 = \frac{6}{35}$

$v^2 = 112{,}4$

$v = 10{,}6$ achteruit dus $v = -10{,}6$



Opgave 15:

a. $\frac{dH}{dt} = 0$ voor $H = 300$

$300a(1 - \frac{300}{b}) = 0$

$a = 0 \quad \vee \quad \frac{300}{b} = 1$

$a = 0 \quad \vee \quad b = 300$

$a = 0$ vervalt want dan is $\frac{dH}{dt} = 0$ en dus kan de zonnebloem niet groeien.

$a < 0$ vervalt want dan is $\frac{dH}{dt} < 0$ dus wordt ze zonnebloem korter dus $a > 0 \quad \wedge \quad b = 300$

b. $10a(1 - \frac{10}{b}) = 0{,}5$ en $50a(1 - \frac{50}{b}) = 3$

$10a - \frac{100a}{b} = 0{,}5$ en $50a - \frac{2500a}{b} = 3$

$250a - \frac{2500a}{b} = 12{,}5$ en $50a - \frac{2500a}{b} = 3$

$\frac{2500a}{b} = 250a - 12{,}5$

dus $50a - 250a + 12{,}5 = 3$

$-200a = -9{,}5$

$a = 0{,}0475$

$\frac{118{,}75}{b} = 11{,}875 - 12{,}5$

$\frac{118{,}75}{b} = -0{,}625$

$b = -190$

15.2 Differentiaalvergelijkingen

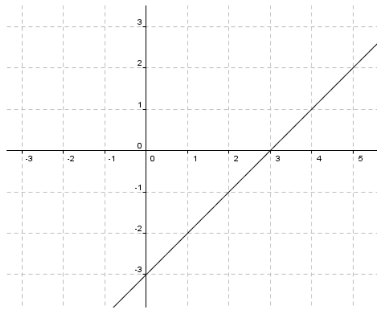
<u>Opgave 16:</u>

a. $\left[\frac{dy}{dt}\right]_{(0,2)} = -5$

$\left[\frac{dy}{dt}\right]_{(4,1)} = 0$

$\left[\frac{dy}{dt}\right]_{(-2,2)} = -7$

$\left[\frac{dy}{dt}\right]_{(4,-1)} = 2$



b. $-y + t - 3 = 0$

$-y = -t + 3$

$y = t - 3$

c. $\frac{dy}{dt} = -y + t - 3 < 0$

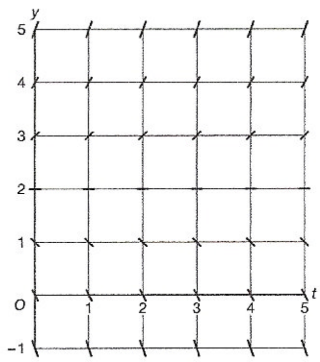
$-y < -t + 3$

$y > t - 3$ dus alle punten boven de bij opgave b getekende lijn

<u>Opgave 17:</u>

a.

y	-1	0	1	2	3	4	5
$\frac{dy}{dt}$	-3	-2	-1	0	1	2	3



b. $\left[\frac{dy}{dt}\right]_{(-3,2)} = 0$

$y = c$ door $(-3,2)$

$c = 2$ dus $y = 2$

c. $\frac{dy}{dt} = y - 2 = 3$

$y = 5$

$5 = 3t - 7$

$-3t = -12$

$t = 4$ dus $A(4,5)$

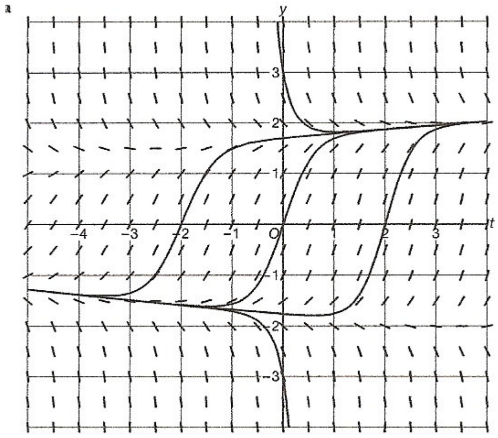
<u>Opgave 18:</u>

$\frac{dy}{dt} = 0$ voor $y = 0 \quad \wedge \quad y = 4$ dus I valt af

als $0 < y < 4$ dan $\frac{dy}{dt} < 0$ dus dit geldt alleen voor III

<u>Opgave 19:</u>

a.



b. $\frac{dy}{dt} = \frac{3}{10}t - y^2 + 3 = 3$

$\frac{3}{10}t - y^2 = 0$ maar ook geldt $y = 3t$ dus

$\frac{3}{10}t - 9t^2 = 0$

$t(\frac{3}{10} - 9t) = 0$

$t = 0 \quad \vee \quad 9t = \frac{3}{10}$

$t = 0 \quad \vee \quad t = \frac{1}{30}$

$y = 0 \quad \vee \quad y = \frac{1}{10}$

$A(0,0)$ of $A(\frac{1}{30}, \frac{1}{10})$

c. als $t = 4$ dan $\frac{dy}{dt} = 0$

$1{,}2 - y^2 + 3 = 0$

$y^2 = 4{,}2$

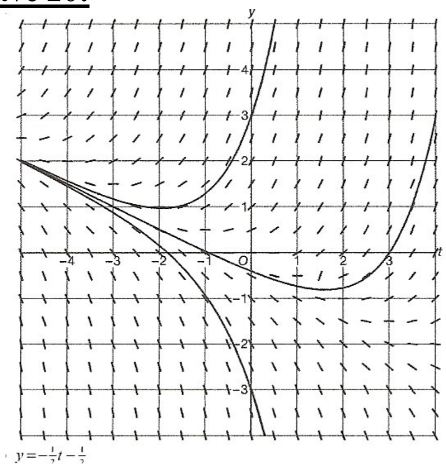
$y = \sqrt{4{,}2} \quad \vee \quad y = -\sqrt{4{,}2}$

d. $\frac{dy}{dt} = \frac{3}{10}t - y^2 + 3 = 0$

$y^2 = \frac{3}{10}t + 3$

Opgave 20:

a.



b. dat is de lijn door $(-1,0)$ en $(1,-1)$ dus

$y = -\frac{1}{2}t - \frac{1}{2}$

c. $\frac{dy}{dt} = \frac{1}{2}t + y = 2$ en er geldt ook $y = 2t + 7$

$\frac{1}{2}t + 2t + 7 = 2$

$2\frac{1}{2}t = -5$

$t = -2$

$y = 3$

$A(-2,3)$

Opgave 21:

$$\frac{dy}{dt} = \frac{t+y}{t-y} = 0$$

$t + y = 0$

dus $y = -t$

Opgave 22:

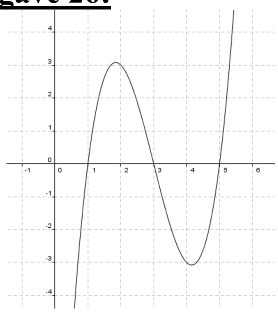
Opgave 23:

Opgave 24:

Opgave 25:

Opgave 26:

a.



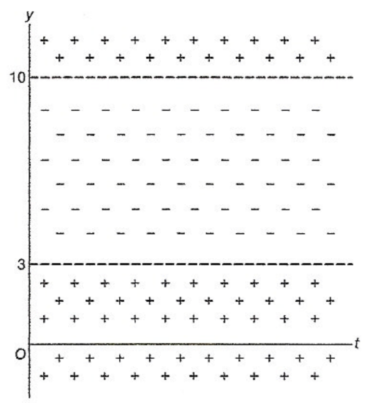
b. als $1 < y < 3$ dan ligt de grafiek boven de horizontale as dus $f(y) > 0$
dus $\frac{dy}{dt} > 0$

c. $y < 1 \quad \vee \quad 3 < y < 5$

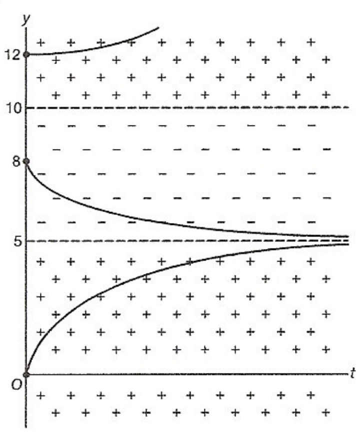
d. $y = 3$ invullen in de dv geeft: $\frac{dy}{dt} = 0$
$y = 3$ differentiëren geeft: $\frac{dy}{dt} = 0$
beide afgeleiden zijn gelijk, dus $y = 3$ is een oplossingskromme

Opgave 27:

a. $f_3(y) = 0$ geeft $(1 - \frac{y}{10})(1 - \frac{y}{3}) = 0$
$1 - \frac{y}{10} = 0 \quad \vee \quad 1 - \frac{y}{3} = 0$
$-\frac{y}{10} = -1 \quad \vee \quad -\frac{y}{3} = -1$
$y = 10 \quad \vee \quad y = 3$
$y = 3$ is horizontale asymptoot als:
$y(0) < 3 \quad \vee \quad 3 < y(0) < 10$

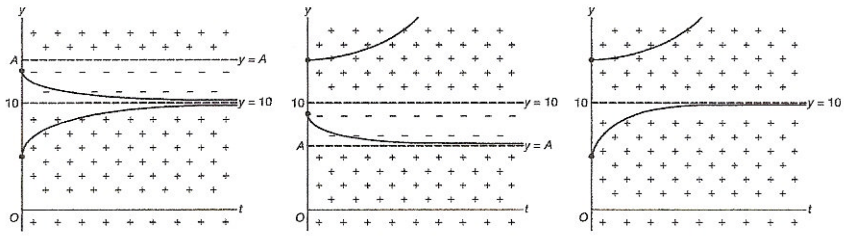


b. $y = 10$ voldoet aan de differentiaalvergelijking
dus $\frac{dy}{dt} = (1 - \frac{10}{10})(1 - \frac{10}{A}) = 0$ voor elke A met $A \neq 0$
$y = 10$ is niet stijgend
dus er zijn geen waarden van A waarvoor alle oplossingskrommen stijgend zijn



c. $f_5(y) = (1 - \frac{y}{10})(1 - \frac{y}{5}) = 0$
$1 - \frac{y}{10} = 0 \quad \vee \quad 1 - \frac{y}{5} = 0$
$y = 10 \quad \vee \quad y = 5$

d.



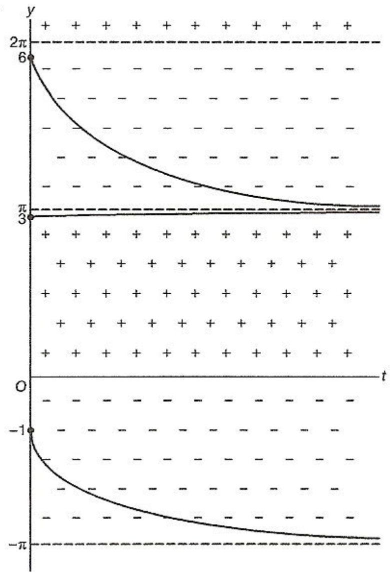
$A > 10$ — $y = 10$ is wel HA

$A < 10$ — $y = 10$ is geen HA

$A = 10$ — $y = 10$ is wel asymptoot

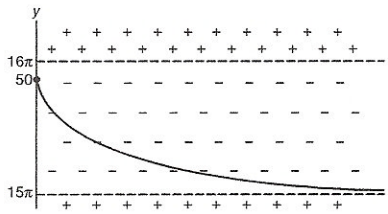
Opgave 28:

a.



b. $15\pi = 47{,}1$ en $16\pi = 50{,}3$

het tekenoverzicht tussen 15π en 16π
is hetzelfde als tussen π en 2π
dus de oplossingskromme daalt
dus $y = 15\pi$ is H.A.



c. $\left[\frac{dy}{dt}\right]_{(4,\frac{1}{6}\pi)} = \frac{1}{2}$

$y = \frac{1}{2}t + b$ door $(4, \frac{1}{6}\pi)$

$\frac{1}{6}\pi = 2 + b$

$b = \frac{1}{6}\pi - 2$

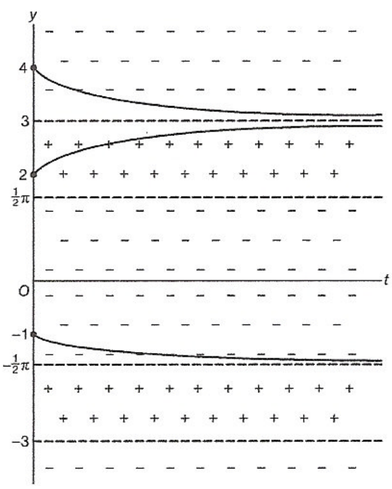
$y = \frac{1}{2}t + \frac{1}{6}\pi - 2$

Opgave 29:

a. $(y^2 - 9)\cos y = 0$

$y^2 = 9 \quad \vee \quad \cos y = 0$

$y = 3 \quad \vee \quad y = -3 \quad \vee \quad y = \frac{1}{2}\pi + k \cdot \pi$



b. $\left[\frac{dy}{dt}\right]_{(6,0)} = -9$

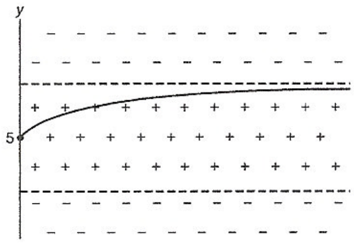
$y = -9t + b$ door $(6,0)$

$0 = -54 + b$

$b = 54$
$y = -9t + 54$

c. als $-3 < y(0) < -\frac{1}{2}\pi \quad \vee \quad -\frac{1}{2}\pi < y(0) < \frac{1}{2}\pi$

Opgave 30:



a. als de oplossingskromme stijgend is dan moet je het hiernaast getekende tekenoverzicht hebben
dus $f_p(5) > 0$ f
$-25 + 5p - 3 > 0$
$5p > 28$
$p > 5{,}6$

b. $\left[\frac{dy}{dt}\right]_{(6,16)} = -256 + 16p - 3 = 16p - 259 = 2$
$16p = 261$
$p = 16\frac{3}{16}$

c. de grafiek van $f_p(y) = -y^2 + py - 3$ is een bergparabool, dus voor elke waarde van p zijn er waarden van y waarvoor geldt: $\frac{dy}{dt} < 0$
dus er is geen waarde van p waarvoor alle oplossingskrommen stijgend zijn

§15.3 Oplossingen van differentiaalvergelijkingen

<u>Opgave 31:</u>

a. $\frac{dy}{dt} = 2(at+b)+t-3$ en $\frac{dy}{dt} = a$

$\frac{dy}{dt} = 2at+2b+t-3$ en $\frac{dy}{dt} = a$

$\frac{dy}{dt} = (2a+1)t+2b-3$ en $\frac{dy}{dt} = a$

b. dus $\begin{cases} 2a+1=0 \\ 2b-3=a \end{cases}$

$2a=-1$

$a=-\frac{1}{2}$

$2b-3=-\frac{1}{2}$

$2b=2\frac{1}{2}$

$b=1\frac{1}{4}$

c. $y=e^{2t}-\frac{1}{2}t+1\frac{1}{4}$

invullen in de dv geeft: $\frac{dy}{dt} = 2(e^{2t}-\frac{1}{2}t+1\frac{1}{4})+t-3$

$= 2e^{2t}-t+2\frac{1}{2}+t-3$

$= 2e^{2t}-\frac{1}{2}$

de oplossing differentiëren geeft: $\frac{dy}{dt} = 2e^{2t}-\frac{1}{2}$ dus het klopt

<u>Opgave 32:</u>

$y=at+b$ differentiëren geeft: $\frac{dy}{dt}=a$

$y=at+b$ ivullen in de dv geeft: $\frac{dy}{dt}=t^2-t(at+b)+\frac{1}{4}(at+b)^2-\frac{1}{4}$

dus $\frac{dy}{dt}=t^2-at^2-bt+\frac{1}{4}a^2t^2+\frac{1}{2}abt+\frac{1}{4}b^2-\frac{1}{4}$

$=(1-a+\frac{1}{4}a^2)t^2+(b+\frac{1}{2}ab)t+\frac{1}{4}b^2-\frac{1}{4}$

$\begin{cases} 1-a+\frac{1}{4}a^2=0 \\ -b+\frac{1}{2}ab=0 \\ \frac{1}{4}b^2-\frac{1}{4}=a \end{cases}$

$a^2-4a+4=0$

$(a-2)^2=0$

$a=2$

$\frac{1}{4}b^2-\frac{1}{4}=2$

$\frac{1}{4}b^2=2\frac{1}{4}$

$b^2=9$

$b=3 \quad \vee \quad b=-3$

dus $y=2t+3$ en $y=2t-3$

Opgave 33:

als $y_1(t) = \frac{36}{t^2}$ dan $\frac{dy_1}{dt} = -\frac{72}{t^3}$

invullen in D_3 geeft: $\frac{dy}{dt} = \frac{-2 \cdot \frac{36}{t^2}}{t} = \frac{-\frac{72}{t^2}}{t} = -\frac{72}{t^3}$

als $y_2(t) = 3e^t + t^2 + 1$ dan $\frac{dy}{dt} = 3e^t + 2t$

invullen in D_1 geeft: $\frac{dy}{dt} = 3e^t + t^2 + 1 - t^2 + 2t - 1 = 3e^t + 2t$

als $y_3(t) = -e^t - 1$ dan $\frac{dy}{dt} = -e^t$

invullen in D_2 geeft: $\frac{dy}{dt} = -e^t - 1 + 1 = -e^t$

Opgave 34:

a. $y = b$

differentiëren geeft: $\frac{dy}{dt} = 0$

invullen geeft: $\frac{dy}{dt} = b - \frac{1}{4}b^2 = 0$

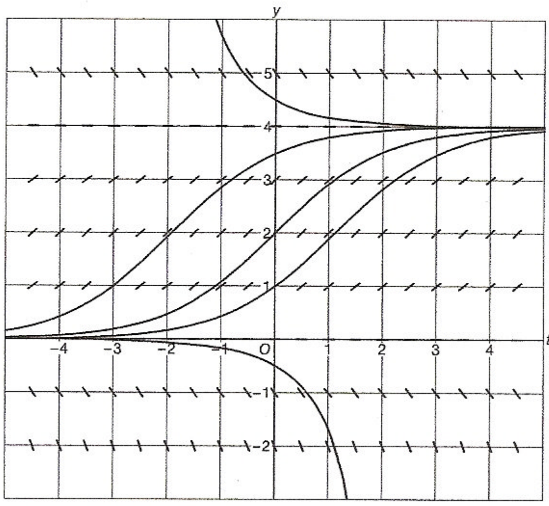
$b(1 - \frac{1}{4}b) = 0$

$b = 0 \quad \vee \quad 1 - \frac{1}{4}b = 0$

$b = 0 \quad \vee \quad b = 4$

dus de oplossingen zijn: $y = 0$ en $y = 4$

b.



c. $y = \frac{4}{1 + e^{-t}}$

invullen geeft: $\frac{dy}{dt} = \frac{4}{1 + e^{-t}} - \frac{1}{4} \cdot \frac{16}{(1 + e^{-t})^2} = \frac{4(1 + e^{-t})}{(1 + e^{-t})^2} - \frac{4}{(1 + e^{-t})^2} = \frac{4e^{-t}}{(1 + e^{-t})^2}$

differentiëren geeft: $\frac{dy}{dt} = \frac{0 - 4 \cdot -e^{-t}}{(1 + e^{-t})^2} = \frac{4e^{-t}}{(1 + e^{-t})^2}$ dus klopt

Opgave 35:

a. $y = at + b$

differentiëren geeft: $\frac{dy}{dt} = a$

invullen geeft: $\frac{dy}{dt} = \frac{1}{2}t + at + b + 1 = (\frac{1}{2} + a)t + b + 1$

$$\begin{cases} \frac{1}{2} + a = 0 \\ b + 1 = a \end{cases}$$
$a = -\frac{1}{2}$
$b + 1 = -\frac{1}{2}$
$b = -1\frac{1}{2}$
$y = -\frac{1}{2}t - 1\frac{1}{2}$

b. invullen geeft: $\frac{dy}{dt} = \frac{1}{2}t + c \cdot e^t - \frac{1}{2}t - 1\frac{1}{2} + 1 = c \cdot e^t - \frac{1}{2}$
differentiëren geeft: $\frac{dy}{dt} = c \cdot e^t - \frac{1}{2}$ dus klopt

c. t-as raken dus $\frac{dy}{dt} = 0 \quad \wedge \quad y = 0$
$\frac{dy}{dt} = 0$ geeft $c \cdot e^t - \frac{1}{2} = 0$
$c \cdot e^t = \frac{1}{2}$
$y = 0$ geeft $\frac{1}{2} - \frac{1}{2}t - 1\frac{1}{2} = 0$
$-\frac{1}{2}t = 1$
$t = -2$
$c \cdot e^{-2} = \frac{1}{2}$
$c = \frac{1}{2}e^2$

Opgave 36:

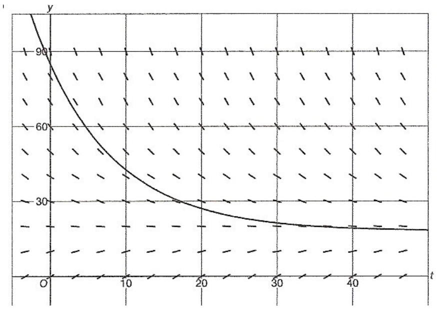
a. $M(2,-1)$

b. $(t-2)^2 + (y+1)^2 = 18$
$2(t-2)dt + 2(y+1)dy = 0$
$2(y+1)dy = -2(t-2)dt$
$$\frac{dy}{dt} = \frac{-(t-2)}{y+1} = \frac{2-t}{y+1}$$

c. $r = \sqrt{2^2 + 7^2} = \sqrt{53}$
$(t-2)^2 + (y+1)^2 = 53$
$2(t-2)dt + 2(y+1)dy = 0$
$2(y+1)dy = -2(t-2)dt$
$$\frac{dy}{dt} = \frac{-(t-2)}{y+1} = \frac{2-t}{y+1}$$

Opgave 37:

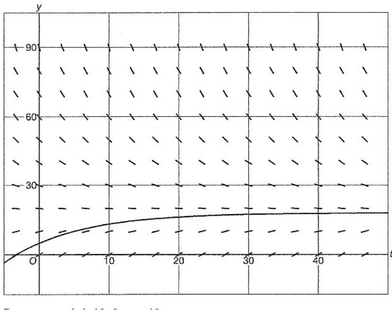
a. $\frac{dT}{dt} = -\frac{1}{10}(T - 18) < 0$
$T - 18 > 0$
$T > 18$

b.


c. $T = 18 + 67e^{-0,1t}$

invullen geeft: $\frac{dT}{dt} = -0,1 \cdot (18 + 67e^{-0,1t} - 18) = -6,7e^{-0,1t}$

differentiëren geeft: $\frac{dT}{dt} = 67 \cdot -0,1e^{-0,1t} = -6,7e^{-0,1t}$

d. 

e. differentiëren geeft: $\frac{dT}{dt} = q \cdot -0,1e^{-0,1t} = -0,1q \cdot e^{-0,1t}$

invullen geeft: $\frac{dT}{dt} = -0,1 \cdot (p + q \cdot e^{-0,1t} - 18) = -0,1p + 1,8 - 0,1q \cdot e^{-0,1t}$

$$\begin{cases} -0,1p + 1,8 = 0 \\ -0,1q = -0,1q \end{cases}$$

$-0,1p = -1,8$

$p = 18$

$T(t) = 18 + q \cdot e^{-0,1t}$

$T(0) = 18 + q = 5$

$q = -13$

Opgave 38:

$y_1(t) = 1\frac{1}{2}t^2 + 5$ dus $\frac{dy_1}{dt} = 3t$

$y_2(t) = 5e^{3t}$ dus $\frac{dy_2}{dt} = 5 \cdot 3e^{3t} = 15e^{3t}$

$y_3(t) = 3t^2$ dus $\frac{dy_3}{st} = 6t$

$y_4(t) = e^{3t} + 2$ dus $\frac{dy_4}{dt} = 3e^{3t}$

$y_5(t) = 1\frac{1}{2}t^2 - 6$ dus $\frac{dy_5}{dt} = 3t$

$y_6(t) = -3e^{3t}$ dus $\frac{dy_6}{dt} = -3 \cdot 3e^{3t} = -9e^{3t}$

a. y_1 en y_5 zijn oplossingen van D_1

b. y_2 en y_6 zijn oplossingen van D_2

Opgave 39:

a. $y(t) = G(t) + c$

differentiëren geeft: $\frac{dy}{dt} = G'(t) = g(t)$

invullen geeft: $\frac{dy}{dt} = g(t)$ dus klopt

b. $y(t) = c \cdot e^{at}$

differentiëren geeft: $\frac{dy}{dt} = c \cdot ae^{at} = a \cdot ce^{at}$

invullen geeft: $\frac{dy}{dt} = a \cdot ce^{at}$ dus klopt

Opgave 40:

a. $\frac{dy}{dt} = \cos(t) + t^2 + 5$

$y(t) = \sin(t) + \frac{1}{3}t^3 + 5t + c$

b. $\frac{dy}{dt} = 0{,}2y$

$\frac{1}{y}dy = 0{,}2dt$

$\ln|y| = 0{,}2t + c_1$

$|y| = e^{0{,}2t + c_1} = e^{0{,}2t} \cdot e^{c_1} = c_2 \cdot e^{0{,}2t}$

$y = c \cdot e^{0{,}2t}$

c. $\frac{dy}{dt} = \frac{5}{y^2}$

$y^2 dy = 5dt$

$\frac{1}{3}y^3 = 5t + c_1$

$y^3 = 15t + c$

d. $\frac{dy}{dt} = \frac{1-t}{y-3}$

$(y-3)dy = (1-t)dt$

$\frac{1}{2}(y-3)^2 = \frac{1}{2}(1-t)^2 \cdot -1 + c_1$

$\frac{1}{2}(1-t)^2 + \frac{1}{2}(y-3)^2 = c_1$

$(1-t)^2 + (y-3)^2 = c$

Opgave 41:

a. $\frac{dy}{dt} = \frac{6t}{y+1}$

$(y+1)dy = 6tdt$

$\frac{1}{2}(y+1)^2 = 3t^2 + c_1$

$(y+1)^2 = 6t^2 + c$

b. $\frac{dy}{dt} = \frac{1}{t \cdot y^2}$

$y^2 dy = \frac{1}{t}dt$

$\frac{1}{3}y^3 = \ln|t| + c_1$

$y^3 = 3\ln|t| + c$

c. $\frac{dy}{dt} = t^2 + t^2 y$

$\frac{dy}{dt} = t^2(1+y)$

$\frac{1}{y+1}dy = t^2 dt$

$\ln(y+1) = \frac{1}{3}t^3 + c_1$

$y+1=e^{\frac{1}{3}t^3+c_1}=e^{\frac{1}{3}t^3}\cdot e^{c_1}=c_2\cdot e^{\frac{1}{3}t^3}$

$y=-1+c\cdot e^{\frac{1}{3}t^3}$

d. $\frac{dy}{dt}=(1+y)e^t$

$\frac{1}{y+1}dy=e^t dt$

$\ln(y+1)=e^t+c_1$

$y+1=e^{e^t+c_1}=e^{e^t}\cdot e^{c_1}=c_2\cdot e^{e^t}$

$y=-1+c\cdot e^{e^t}$

Opgave 42:

a. $I(0)=5e^0=5$

differentiëren geeft: $\frac{dI}{dt}=5\cdot -e^{-t}=-5e^{-t}$

invullen geeft: $\frac{dI}{dt}=\frac{-1}{R\cdot c}\cdot I=\frac{-1}{R\cdot c}\cdot 5e^{-t}=\frac{-5e^{-t}}{R\cdot c}$

dus $R\cdot c=1$

b. $\frac{dI}{dt}=\frac{-1}{10}\cdot I=-0{,}1I$

$\frac{1}{I}\cdot dI=-0{,}1dt$

$\ln I=-{,}01t+c_1$

$I=e^{-0{,}1t+c_1}=c\cdot e^{-0{,}1t}$

$I(2)=c\cdot e^{-0{,}2}=3$

$c=3e^{0{,}2}=3\cdot\sqrt[5]{e}$

$I(0)=c\cdot e^0=c$ dus $I(0)=3\cdot\sqrt[5]{e}$

15.4 Differentiaalvergelijkingen van de eerste orde

Opgave 43:

a. $y = at^2 + bt + c$

differentiëren geeft: $\frac{dy}{dt} = 2at + b$

invullen geeft: $\frac{dy}{dt} = 2(at^2 + bt + c) - 4t^2 + 4t$

$= 2at^2 + 2bt + 2c - 4t^2 + 4t$

$= (2a-4)t^2 + (2b+4)t + 2c$

$$\begin{cases} 2a - 4 = 0 \\ 2a = 2b + 4 \\ b = 2c \end{cases}$$

$2a = 4$

$a = 2$

$4 = 2b + 4$

$b = 0$

$c = 0$

$y = 2t^2$

b. $\frac{dy}{dt} = 2y$

$y = c \cdot e^{2t}$

c. $y = c \cdot e^{2t} + 2t^2$

differentiëren geeft: $\frac{dy}{dt} = 2c \cdot e^{2t} + 4t$

invullen geeft: $\frac{dy}{dt} = 2(c \cdot e^{2t} + 2t^2) - 4t^2 + 4t$

$= 2c \cdot e^{2t} + 4t^2 - 4t^2 + 4t$

$= 2c \cdot e^{2t} + 4t$ dus klopt

Opgave 44:

a. $y = a(t) + p(t)$

differentiëren geeft: $\frac{dy}{dt} = a'(t) + p'(t) = f(t) \cdot dt + f(t) \cdot p(t) + g(t)$

invullen geeft: $\frac{dy}{dt} = f(t) \cdot (a(t) + p(t)) + g(t) = f(t) \cdot a(t) + f(t) \cdot p(t) + g(t)$ klopt

b. $\frac{dy}{dt} = f(t) \cdot y$

$a(t) = c \cdot e^{F(t)}$

differentiëren geeft: $\frac{dy}{dt} = f(t) \cdot c \cdot e^{F(t)}$

invullen geeft: $\frac{dy}{dt} = f(t) \cdot c \cdot e^{F(t)}$ dus klopt

c. $y = a(t) + p(t) = c \cdot e^{F(t)} + p(t)$

$c \cdot e^{F(t)} = y - p(t)$

$c = \frac{y - p(t)}{e^{F(t)}}$

voor elk punt (t, y) bestaat er precies één waarde van c dus door elk punt (t, y) gaat er precies één oplossingskromme.

Opgave 45:

a. $\frac{dy}{dt} = -\frac{1}{2}y + 2t - 4$

$\frac{dy}{dt} = -\frac{1}{2}y$ geeft $y = c \cdot e^{-\frac{1}{2}t}$

stel $y = at + b$

differentiëren geeft: $\frac{dy}{dt} = a$

invullen geeft: $\frac{dy}{dt} = -\frac{1}{2}(at + b) + 2t - 4$

$= -\frac{1}{2}at - \frac{1}{2}b + 2t - 4$

$= (-\frac{1}{2}a + 2)t - \frac{1}{2}b - 4$

$\begin{cases} -\frac{1}{2}a + 2 = 0 \\ -\frac{1}{2}b - 4 = a \end{cases}$

$-\frac{1}{2}a = -2$

$a = 4$

$-\frac{1}{2}b - 4 = 4$

$-\frac{1}{2}b = 8$

$b = -16$

$y = 4t - 16$ is een particuliere oplossing

dus $y = c \cdot e^{-\frac{1}{2}t} + 4t - 16$

b. $\frac{dy}{dt} = 6 - 2y$

$y = 3$ is een particuliere oplossing

$\frac{dy}{dt} = -2y$ geeft $y = c \cdot e^{-2t}$

dus $y = c \cdot e^{-2t} + 3$

c. $\frac{dy}{dt} = 2y - 4t + 6$

$\frac{dy}{dt} = 2y$ geeft $y = c \cdot e^{2t}$

stel $y = at + b$

differentiëren geeft: $\frac{dy}{dt} = a$

invullen geeft: $\frac{dy}{dt} = 2(at + b) - 4t + 6$

$= 2at + 2b - 4t + 6$

$= (2a - 4)t + 2b + 6$

$\begin{cases} 2a - 4 = 0 \\ 2b + 6 = a \end{cases}$

$2a = 4$

$a = 2$

$2b + 6 = 2$

$2b = -4$

$b = -2$

$y = 2t - 2$ is een particuliere oplossing

dus $y = c \cdot e^{2t} + 2t - 2$

d. $\frac{dT}{dt} = -\frac{1}{10}(T - 21)$

$T = 21$ is een particuliere oplossing

$\frac{dT}{dt} = -\frac{1}{10}T$ geeft $T = c \cdot e^{-\frac{1}{10}t}$

dus $T = c \cdot e^{-\frac{1}{10}t} + 21$

Opgave 46:

a. $\frac{dy}{dt} = 2y + \cos t$

$\frac{dy}{dt} = 2y$ geeft $y = c \cdot e^{2t}$

stel $y = A\sin t + B\cos t$

differentiëren geeft: $\frac{dy}{dt} = A\cos t - B\sin t$

invullen geeft: $\frac{dy}{dt} = 2(A\sin t + B\cos t) + \cos t$

$= 2A\sin t + 2B\cos t + \cos t$

$= 2A\sin t + (2B+1)\cos t$

$\begin{cases} 2A = B \\ 2B+1 = A \end{cases}$

$2(2B+1) = -B$

$4B + 2 = -B$

$5B = -2$

$B = -\frac{2}{5}$

$A = \frac{1}{5}$

$y = \frac{1}{5}\sin t - \frac{2}{5}\cos t$ is een particuliere oplossing

dus $y = c \cdot e^{2t} + \frac{1}{5}\sin t - \frac{2}{5}\cos t$

b. $\frac{dy}{dt} = -3y - \sin t$

$\frac{dy}{dt} = -3y$ geeft $y = c \cdot e^{-3t}$

stel $y = A\sin t + B\cos t$

differentiëren geeft: $\frac{dy}{dt} = A\cos t - B\sin t$

invullen geeft: $\frac{dy}{dt} = -3(a\sin t + B\cos t) - \sin t$

$= -3A\sin t - 3B\cos t - \sin t$

$= (-3A-1)\sin t - 3B\cos t$

$\begin{cases} -3A - 1 = -B \\ -3B = A \end{cases}$

$-3 \cdot -3B - 1 = -B$

$9B - 1 = -B$

$10B = 1$

$B = \frac{1}{10}$

$A = -\frac{3}{10}$

$y = -\frac{3}{10}\sin t + \frac{1}{10}\cos t$ is een particuliere oplossing

dus $y = c \cdot e^{-3t} - \frac{3}{10}\sin t + \frac{1}{10}\cos t$

c. $\frac{dy}{dt} = 4y - \cos t$

$\frac{dy}{dt} = 4y$ geeft $y = c \cdot e^{4t}$

stel $y = A\sin t + B\cos t$

differentiëren geeft: $\frac{dy}{dt} = A\cos t - B\sin t$

invullen geeft: $\frac{dy}{dt} = 4(A\sin t + B\cos t) - \cos t$

$= 4A\sin t + 4B\cos t - \cos t$
$= 4A\sin t + (4B-1)\cos t$

$\begin{cases} 4A = -B \\ 4B - 1 = A \end{cases}$

$4(4B-1) = -B$
$16B - 4 = -B$
$17B = 4$
$B = \frac{4}{17}$
$A = -\frac{1}{17}$
$y = -\frac{1}{17}\sin t + \frac{4}{17}\cos t$ is een particuliere oplossing
dus $y = c \cdot e^{4t} - \frac{1}{17}\sin t + \frac{4}{17}\cos t$

d. $\frac{dy}{dt} = \frac{1}{2}y + 2\sin t - 3\cos t$
$\frac{dy}{dt} = \frac{1}{2}y$ geeft $y = c \cdot e^{\frac{1}{2}t}$
stel $y = A\sin t + B\cos t$
differentiëren geeft: $\frac{dy}{dt} = A\cos t - B\sin t$
invullen geeft: $\frac{dy}{dt} = \frac{1}{2}(A\sin t + B\cos t) + 2\sin t - 3\cos t$
$= \frac{1}{2}A\sin t + \frac{1}{2}B\cos t + 2\sin t - 3\cos t$
$= (\frac{1}{2}A + 2)\sin t + (\frac{1}{2}B - 3)\cos t$

$\begin{cases} \frac{1}{2}A + 2 = -B \\ \frac{1}{2}B - 3 = A \end{cases}$

$\frac{1}{2}(\frac{1}{2}B - 3) + 2 = -B$
$\frac{1}{4}B - 1\frac{1}{2} + 2 = -B$
$1\frac{1}{4}B = -\frac{1}{2}$
$B = -\frac{2}{5}$
$A = -3\frac{1}{5}$
$y = -3\frac{1}{5}\sin t - \frac{2}{5}\cos t$ is een particuliere oplossing
dus $y = c \cdot e^{\frac{1}{2}t} - 3\frac{1}{5}\sin t - \frac{2}{5}\cos t$

<u>Opgave 47:</u>

a. $\frac{dy}{dt} = 2y - 3e^t$
$\frac{dy}{dt} = 2y$ geeft $y = c \cdot e^{2t}$
stel $y = A \cdot e^t$
differentiëren geeft: $\frac{dy}{dt} = A \cdot e^t$
invullen geeft: $\frac{dy}{dt} = 2A \cdot e^t - 3e^t = (2A-3)e^t$
$2A - 3 = A$
$A = 3$
$y = 3e^t$ is een particuliere oplossing
dus $y = c \cdot e^{2t} + 3e^t$

b. $\frac{dy}{dt} = 10y - 5e^t$

$\frac{dy}{dt} = 10y$ geeft $y = c \cdot e^{10t}$

stel $y = A \cdot e^t$

differentiëren geeft: $\frac{dy}{dt} = A \cdot e^t$

invullen geeft: $\frac{dy}{dt} = 10A \cdot e^t - 5e^t = (10A - 5)e^t$

$10A - 5 = A$
$9A = 5$
$A = \frac{5}{9}$

$y = \frac{5}{9}e^t$ is een particuliere oplossing

dus $y = c \cdot e^{10t} + \frac{5}{9}e^t$

c. $\frac{dy}{dt} = -2y + e^t$

$\frac{dy}{dt} = -2y$ geeft $y = c \cdot e^{-2t}$

stel $y = A \cdot e^t$

differentiëren geeft: $\frac{dy}{dt} = A \cdot e^t$

invullen geeft: $\frac{dy}{dt} = -2A \cdot e^t + e^t = (-2A + 1)e^t$

$-2A + 1 = A$
$-3A = -1$
$A = \frac{1}{3}$

$y = \frac{1}{3}e^t$ is een particuliere oplossing

dus $y = c \cdot e^{-2t} + \frac{1}{3}e^t$

d. $\frac{dy}{dt} = 1\frac{1}{2}y + 2e^t$

$\frac{dy}{dt} = 1\frac{1}{2}y$ geeft $y = c \cdot e^{1\frac{1}{2}t}$

stel $y = A \cdot e^t$

differentiëren geeft: $\frac{dy}{dt} = A \cdot e^t$

invullen geeft: $\frac{dy}{dt} = 1\frac{1}{2}A \cdot e^t + 2e^t = (1\frac{1}{2}A + 2)e^t$

$1\frac{1}{2}A + 2 = A$

$\frac{1}{2}A = -2$

$A = -4$

$y = -4e^t$ is een particuliere oplossing

dus $y = c \cdot e^{1\frac{1}{2}t} - 4e^t$

Opgave 48:

a. $\frac{dy}{dt} = 2y + 4t - 2$

$\frac{dy}{dt} = 2y$ geeft $y = c \cdot e^{2t}$

stel $y = at + b$

differentiëren geeft: $\frac{dy}{dt} = a$

invullen geeft: $\frac{dy}{dt} = 2(at + b) + 4t - 2$
$= 2at + 2b + 4t - 2$
$= (2a + 4)t + 2b - 2$

$$\begin{cases} 2a + 4 = 0 \\ 2b - 2 = a \end{cases}$$

$2a = -4$

$a = -2$

$2b - 2 = -2$

$b = 0$

$y = -2t$ is een particuliere oplossing

dus $y = c \cdot e^{2t} - 2t$

b. $\frac{dy}{dt} = \frac{1}{2}y + 4e^{2t}$

$\frac{dy}{dt} = \frac{1}{2}y$ geeft $y = c \cdot e^{\frac{1}{2}t}$

stel $y = A \cdot e^{2t}$

differentiëren geeft: $\frac{dy}{dt} = 2A \cdot e^{2t}$

invullen geeft: $\frac{dy}{dt} = \frac{1}{2}A \cdot e^{2t} + 4e^{2t} = (\frac{1}{2}A + 4) \cdot e^{2t}$

$\frac{1}{2}A + 4 = 2A$

$-1\frac{1}{2}A = -4$

$A = 2\frac{2}{3}$

$y = 2\frac{2}{3}e^{2t}$ is een particuliere oplossing

dus $y = c \cdot e^{\frac{1}{2}t} + 2\frac{2}{3}e^{2t}$

c. $\frac{dy}{dt} = -3y + \sin(2t)$

$\frac{dy}{dt} = -3y$ geeft $y = c \cdot e^{-3t}$

stel $y = A\sin(2t) + B\cos(2t)$

differentiëren geeft: $\frac{dy}{dt} = 2A\cos(2t) - 2B\sin(2t)$

invullen geeft: $\frac{dy}{dt} = -3(A\sin(2t) + B\cos(2t)) + \sin(2t)$

$= -3A\sin(2t) - 3B\cos(2t) + \sin(2t)$

$= (-3A + 1)\sin(2t) - 3B\cos(2t)$

$$\begin{cases} -3A + 1 = -2B \\ \quad -3B = 2A \end{cases}$$

$A = -1\frac{1}{2}B$

$4\frac{1}{2}B + 1 = -2B$

$6\frac{1}{2}B = -1$

$B = -\frac{2}{13}$

$A = \frac{3}{13}$

$y = \frac{3}{13}\sin(2t) - \frac{2}{13}\cos(2t)$ is een particuliere oplossing

dus $y = c \cdot e^{-3t} + \frac{3}{13}\sin(2t) - \frac{2}{13}\cos(2t)$

d. $\frac{dy}{dt} = -2y + 2t^2 - 4t$

$\frac{dy}{dt} = -2y$ geeft $y = c \cdot e^{-2t}$

stel $y = at^2 + bt + c$

differentiëren geeft: $\frac{dy}{dt} = 2at + b$

invullen geeft: $\frac{dy}{dt} = -2(at^2 + bt + c) + 2t^2 - 4t$

$= -2at^2 - 2bt - 2c + 2t^2 - 4t$
$= (-2a + 2)t^2 + (-2b - 4)t - 2c$

$$\begin{cases} -2a + 2 = 0 \\ -2b - 4 = 2a \\ -2c = b \end{cases}$$

$-2a = -2$
$a = 1$
$-2b - 4 = 2$
$-2b = 6$
$b = -3$
$-2c = -3$
$c = 1\tfrac{1}{2}$
$y = t^2 - 3t + 1\tfrac{1}{2}$ is een particuliere oplossing
dus $y = c \cdot e^{-2t} + t^2 - 3t + 1\tfrac{1}{2}$

Opgave 49:

a. $\frac{dy}{dt} = -y - 3$
$y = -3$ is een particuliere oplossing
$\frac{dy}{dt} = -y$ geeft $\frac{dy}{dt} = c \cdot e^{-t}$
dus $y = c \cdot e^{-t} - 3$
$y(0) = c - 3 = 1$
$c = 4$
$y(t) = 4e^{-t} - 3$

b. $y(0) = c - 3 = 0$
$c = 3$
$y(t) = 3e^{-t} - 3$
$y(1) = 3e^{-1} - 3 = -3 + \frac{3}{e}$ dus het punt $(1, -3 + \frac{3}{e})$ ligt op de oplossingskromme

c. $y(3) = c \cdot e^{-3} - 3 = 1$
$c \cdot e^{-3} = 4$
$c = 4e^3$
$y(t) = 4e^3 \cdot e^{-t} - 3 = 4e^{3-t} - 3$
$y(a) = 4e^{3-a} - 3 = 8$
$4e^{3-a} = 11$
$e^{3-a} = 2\tfrac{3}{4}$
$3 - a = \ln(2\tfrac{3}{4})$
$a = 3 - \ln(2\tfrac{3}{4})$

Opgave 50:

a. $\frac{dT}{dt} = -0{,}035(T - 18)$
$T = 18$ is een particuliere oplossing
$\frac{dT}{dt} = -0{,}035T$ geeft $T = c \cdot e^{-0{,}035t}$
$T = c \cdot e^{-0{,}035t} + 18$

$T(0) = c + 18 = 5$
$c = -13$
$T = -13e^{-0,035t} + 18$

b. $T(5) = -13e^{-0,175} + 18 = 7$ ºC

c. $-13e^{-0,035t} + 18 = 15$
$-13e^{-0,035t} = -3$
$e^{-0,035t} = \frac{3}{13}$
$-0,035t = \ln(\frac{3}{13})$
$t = \frac{\ln(\frac{3}{13})}{-0,035} = 42$

Opgave 51:

a. $\frac{dT}{dt} = -0,03(T - 150)$
$T = 150$ is een particuliere oplossing
$\frac{dT}{dt} = -0,03T$ geeft $T = c \cdot e^{-0,03t}$
$T = c \cdot e^{-0,03t} + 150$
$T(0) = c + 150 = 20$
$c = -130$
$T = -130 \cdot e^{-0,03t} + 150$
$-130e^{-0,03t} + 150 = 100$
$-130e^{-0,03t} = -50$
$e^{-0,03t} = \frac{5}{13}$
$-0,03t = \ln(\frac{5}{13})$
$t = \frac{\ln(\frac{5}{13})}{-0,03} = 31,9$ dus na 32 minuten

b. $\frac{dT}{dt} = -0,001(T - 150)$
$T = 150$ is een particuliere oplossing
$\frac{dT}{dt} = -0,001T$ geeft $T = c \cdot e^{-0,001t}$
$T = c \cdot e^{-0,001t} + 150$
$T(32) = c \cdot e^{-0,032} + 150 = 100$
$c \cdot e^{-0,032} = -50$
$c = -50e^{0,032}$
$T = -50e^{0,032} \cdot e^{-0,001t} + 150 = -50e^{0,032-0,001t} + 150$
$-50e^{0,032-0,001t} + 150 = 120$
$-50e^{0,032-0,001t} = -30$
$e^{0,032-0,001t} = 0,6$
$0,032 - 0,001t = \ln(0,6)$
$-0,001t = \ln(0,6) - 0,032$
$t = 543$
dus het duurt nog ongeveer $543 - 32 = 511$ minuten

Opgave 52:

a. $\frac{dN}{dt} = 0,7N$ geeft $N = c \cdot e^{0,7t}$
$N(0) = c = 1000$

$N = 1000e^{0,7t}$

b. $\left[\frac{dN}{dt}\right]_{N=10000} = 6825$

$\left[\frac{dN}{dt}\right]_{N=200000} = 70000$

$\left[\frac{dN}{dt}\right]_{N=390000} = 6825$

c. $\frac{dN}{dt} = 0,7N \cdot \dfrac{400000 - N}{400000} = \dfrac{0,7N}{400000} \cdot (400000 - N) = 1,75 \cdot 10^{-6} \cdot (400000 - N)$

Opgave 53:

a. $y = \dfrac{1}{u}$ dan $\frac{dy}{du} = -\dfrac{1}{u^2}$

$\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dt} = -\dfrac{1}{u^2} \cdot \frac{du}{dt}$

b. $\frac{dy}{dt} = -\dfrac{1}{u^2} \cdot \frac{du}{dt}$ dus $\frac{du}{dt} = -u^2 \cdot \frac{dy}{dt}$

$y = \dfrac{1}{u}$ dus $u = \dfrac{1}{y}$

invullen geeft: $\frac{du}{dt} = -\dfrac{1}{y^2} \cdot y(1-y) = -\dfrac{1}{y} + 1 = -(\dfrac{1}{y} - 1) = -(u-1)$

$u = 1$ is een particuliere oplossing

$\frac{du}{dt} = -u$ geeft $u = c \cdot e^{-t}$

$u = c \cdot e^{-t} + 1$

c. $y = \dfrac{1}{u} = \dfrac{1}{1 + c \cdot e^{-t}}$

Opgave 54:

a. $\frac{dy}{dt} = y^2 + 4y$

stel $y = \dfrac{1}{u}$ dan $\frac{dy}{du} = -\dfrac{1}{u^2}$

$\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dt} = -\dfrac{1}{u^2} \cdot \frac{du}{dt} = \dfrac{1}{u^2} + \dfrac{4}{u}$

$\frac{du}{dt} = -1 - 4u$

$u = -\frac{1}{4}$ is een particuliere oplossing

$\frac{du}{dt} = -4u$ geeft $u = c \cdot e^{-4t}$

$u = -\frac{1}{4} + c \cdot e^{-4t}$

$y = \dfrac{1}{-\frac{1}{4} + c \cdot e^{-4t}} = \dfrac{-4}{1 + c_1 \cdot e^{-4t}}$

b. $\frac{dy}{dt} = \frac{1}{2}y(1-y) = \frac{1}{2}y - \frac{1}{2}y^2$

stel $y = \dfrac{1}{u}$ dan $\frac{dy}{du} = -\dfrac{1}{u^2}$

$\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dt} = -\dfrac{1}{u^2} \cdot \frac{du}{dt} = \dfrac{1}{2u} - \dfrac{1}{2u^2}$

$\frac{du}{dt} = -\frac{1}{2}u + \frac{1}{2}$

$u = 1$ is een particuliere oplossing

$\frac{du}{dt} = -\frac{1}{2}u$ geeft $u = c \cdot e^{-\frac{1}{2}t}$

$u = 1 + c \cdot e^{-\frac{1}{2}t}$

$$y = \frac{1}{1 + c \cdot e^{-\frac{1}{2}t}}$$

c. $\frac{dy}{dt} = 0{,}1y(1 - \frac{y}{100}) = 0{,}1y - 0{,}001y^2$

stel $y = \frac{1}{u}$ dan $\frac{dy}{du} = -\frac{1}{u^2}$

$\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dt} = -\frac{1}{u^2} \cdot \frac{du}{dt} = \frac{0{,}1}{u} - \frac{0{,}001}{u^2}$

$\frac{du}{dt} = -0{,}1u + 0{,}001$

$u = 0{,}01$ is een particuliere oplossing

$\frac{du}{dt} = -0{,}1u$ geeft $u = c \cdot e^{-0{,}1t}$

$u = 0{,}01 + c \cdot e^{-0{,}1t}$

$$y = \frac{1}{0{,}01 + c \cdot e^{-0{,}1t}} = \frac{100}{1 + c_1 \cdot e^{-0{,}1t}}$$

d. $\frac{dA}{dt} = 0{,}001A(1 - \frac{A}{20}) = 0{,}001A - 0{,}00005A^2$

stel $A = \frac{1}{u}$ dan $\frac{dA}{du} = -\frac{1}{u^2}$

$\frac{dA}{dt} = \frac{dA}{du} \cdot \frac{du}{dt} = -\frac{1}{u^2} \cdot \frac{du}{dt} = 0{,}001 \cdot \frac{1}{u} - 0{,}00005 \cdot \frac{1}{u^2}$

$\frac{du}{dt} = -0{,}001u + 0{,}00005$

$u = 0{,}05$ is een particuliere oplossing

$\frac{du}{dt} = -0{,}001u$ geeft $u = c \cdot e^{-0{,}001t}$

$u = 0{,}05 + c \cdot e^{-0{,}001t}$

$$A = \frac{1}{0{,}05 + c \cdot e^{-0{,}001t}} = \frac{20}{1 + c_1 \cdot e^{-0{,}001t}}$$

Opgave 55:

a. $\frac{dy}{dt} = 2y^2 - 10y$

stel $y = \frac{1}{u}$ dan $\frac{dy}{du} = -\frac{1}{u^2}$

$\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dt} = -\frac{1}{u^2} \cdot \frac{du}{dt} = 2 \cdot \frac{1}{u^2} - 10 \cdot \frac{1}{u}$

$\frac{du}{dt} = -2 + 10u$

$u = \frac{1}{5}$ is een particuliere oplossing

$\frac{du}{dt} = 10u$ geeft $u = c \cdot e^{10t}$

$u = \frac{1}{5} + c \cdot e^{10t}$

$$y = \frac{1}{\frac{1}{5} + c \cdot e^{10t}} = \frac{5}{1 + c_1 \cdot e^{10t}}$$

b. $\frac{dy}{dt} = \frac{1}{2}y + 4e^t - 2\sin t$

$\frac{dy}{dt} = \frac{1}{2}y$ geeft $y = c \cdot e^{\frac{1}{2}t}$

stel $y = C \cdot e^t + A\sin t + B\cos t$

differentiëren geeft: $\frac{dy}{dt} = C \cdot e^t + A\cos t - B\sin t$

invullen geeft: $\frac{dy}{dt} = \frac{1}{2}(C \cdot e^t + A\sin t + B\cos t) + 4e^t - 2\sin t$

$= \frac{1}{2}C \cdot e^t + \frac{1}{2}A\sin t + \frac{1}{2}B\cos t + 4e^t - 2\sin t$

$= (\frac{1}{2}C + 4) \cdot e^t + (\frac{1}{2}A - 2)\sin t + \frac{1}{2}B\cos t$

$$\begin{cases} \frac{1}{2}C + 4 = C \\ \frac{1}{2}A - 2 = -B \\ \frac{1}{2}B = A \end{cases}$$

$-\frac{1}{2}C = -4$

$C = 8$

$\frac{1}{2} \cdot \frac{1}{2}B - 2 = -B$

$1\frac{1}{4}B = 2$

$B = 1\frac{3}{5}$

$A = \frac{4}{5}$

$y = 8e^t + \frac{4}{5}\sin t + 1\frac{3}{5}\cos t$ is een particuliere oplossing

dus $y = c \cdot e^{\frac{1}{2}t} + 8e^t + \frac{4}{5}\sin t + 1\frac{3}{5}\cos t$

c. $\frac{dy}{dt} = 3ty + 6t$

$\frac{dy}{dt} = 3t(y + 2)$

$\frac{1}{y+2}dy = 3tdt$

$\ln(y + 2) = 1\frac{1}{2}t^2 + c$

$y + 2 = e^{1\frac{1}{2}t^2 + c} = c_1 \cdot e^{1\frac{1}{2}t^2}$

$y = -2 + c_1 \cdot e^{1\frac{1}{2}t^2}$

d. $\frac{dy}{dt} = -2y + 12y^2$

stel $y = \frac{1}{u}$ dan $\frac{dy}{du} = -\frac{1}{u^2}$

$\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dt} = -\frac{1}{u^2} \cdot \frac{du}{dt} = -2 \cdot \frac{1}{u} + 12 \cdot \frac{1}{u^2}$

$\frac{du}{dt} = 2u - 12$

$u = 6$ is een particuliere oplossing

$\frac{du}{dt} = 2u$ geeft $u = c \cdot e^{2t}$

$u = 6 + c \cdot e^{2t}$

$$y = \frac{1}{6 + c \cdot e^{2t}}$$

Opgave 56:

a. $\frac{dy}{dt} = y - 2y^2$

stel $y = \dfrac{1}{u}$ dan $\frac{dy}{du} = -\dfrac{1}{u^2}$

$\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dt} = -\dfrac{1}{u^2} \cdot \frac{du}{dt} = \dfrac{1}{u} - \dfrac{2}{u^2}$

$\frac{du}{dt} = -u + 2$

$u = 2$ is een particuliere oplossing

$\frac{du}{dt} = -u$ geeft $u = c \cdot e^{-t}$

$u = 2 + c \cdot e^{-t}$

$y = \dfrac{1}{2 + c \cdot e^{-t}}$

$y(0) = \dfrac{1}{2 + c} = \frac{1}{5}$

$c = 3$

$y = \dfrac{1}{2 + 3e^{-t}}$

b. $y(0) = \dfrac{1}{2 + c} = 2$

$2(2 + c) = 1$

$4 + 2c = 1$

$2c = -3$

$c = -1\frac{1}{2}$

$y = \dfrac{1}{2 - 1\frac{1}{2} e^{-t}}$

$y(1) = \dfrac{1}{2 - 1\frac{1}{2} e^{-1}} = \dfrac{e}{2e - 1\frac{1}{2}} = \dfrac{2e}{4e - 3}$

Opgave 57:

a. $\frac{dN}{dt} = c \cdot N \cdot (10000 - N)$

$\left[\frac{dN}{dt}\right]_{N=800} = 800c(10000 - 800) = 350$

$c = 4{,}755 \cdot 10^{-5}$

$\frac{dN}{dt} = 4.755 \cdot 10^{-5} \cdot N \cdot (10000 - N)$

$= 0{,}4755N - 4{,}755 \cdot 10^{-5} \cdot N^2$

stel $N = \dfrac{1}{u}$ dan $\frac{dN}{du} = -\dfrac{1}{u^2}$

$\frac{dN}{dt} = \frac{dN}{du} \cdot \frac{du}{dt} = -\dfrac{1}{u^2} \cdot \frac{du}{dt} = 0{,}4755 \cdot \dfrac{1}{u} - 4{,}755 \cdot 10^{-5} \cdot \dfrac{1}{u^2}$

$\frac{du}{dt} = -0{,}4755u + 4{,}755 \cdot 10^{-5}$

$u = 0{,}0001$ is een particuliere oplossing

$\frac{du}{dt} = -0{,}4755u$ geeft $u = c \cdot e^{-0{,}4755t}$

$u = 0{,}0001 + c \cdot e^{-0{,}4755t}$

$$y = \frac{1}{0{,}0001 + c \cdot e^{-0{,}4755t}} = \frac{10000}{1 + c_1 \cdot e^{-0{,}4755t}}$$

$$y(0) = \frac{10000}{1 + c} = 800$$

$800 + 800c = 10000$

$800c = 9200$

$c = 11{,}5$

$$y = \frac{10000}{1 + 11{,}5e^{-0{,}4755t}}$$

$y(20) = 9991$

b. $$y = \frac{10000}{1 + 11{,}5e^{-0{,}4755t}} = 5000$$

neem $y_1 = \dfrac{10000}{1 + 11{,}5e^{-0{,}4755x}}$ en $y_2 = 5000$

intersect geeft $x = 5{,}14$

dus vanaf $t = 5{,}14$ zijn er meer dan 5000 zeeotters

Opgave 58:

a. $\frac{dL}{dt} = 6L - 0{,}2L^2$

stel $L = \dfrac{1}{u}$ dan $\frac{dL}{du} = -\dfrac{1}{u^2}$

$$\tfrac{dL}{dt} = \tfrac{dL}{du} \cdot \tfrac{du}{dt} = -\frac{1}{u^2} \cdot \tfrac{du}{dt} = \frac{6}{u} - 0{,}2 \cdot \frac{1}{u^2}$$

$\frac{du}{dt} = -6u + 0{,}2$

$u = \frac{1}{30}$ is een particuliere oplossing

$\frac{du}{dt} = -6u$ geeft $u = c \cdot e^{-6t}$

$u = \frac{1}{30} + c \cdot e^{-6t}$

$$L = \frac{1}{\frac{1}{30} + c \cdot e^{-t}} = \frac{30}{1 + c_1 \cdot e^{-6t}}$$

$$L(0) = \frac{30}{1 + c_1} = 2$$

$c_1 = 14$

$$L = \frac{30}{1 + 14e^{-6t}}$$

$L(\frac{1}{7}) - L(0) = 4{,}32 - 2 = 2{,}32$ cm

b. de grenswaarde is 30 cm

$$L = \frac{30}{1 + 14e^{-6t}} = 29$$

neem $y_1 = \dfrac{30}{1 + 14e^{-6x}}$ en $y_2 = 29$

intersect geeft $x = 1{,}01$ dus na 7 dagen

Opgave 59:

a. $\frac{dN}{dt} = 0{,}00006N(5000 - N)$

$= 0{,}3N - 0{,}00006N^2$

stel $N = \frac{1}{u}$ dan $\frac{dN}{du} = -\frac{1}{u^2}$

$\frac{dN}{dt} = \frac{dN}{du} \cdot \frac{du}{dt} = -\frac{1}{u^2} \cdot \frac{du}{dt} = 0{,}3 \cdot \frac{1}{u} - 0{,}00006 \cdot \frac{1}{u^2}$

$\frac{du}{dt} = -0{,}3u + 0{,}00006$

$u = 0{,}0002$ is een particuliere oplossing

$\frac{du}{dt} = -0{,}3u$ geeft $u = c \cdot e^{-0{,}3t}$

$u = 0{,}0002 + c \cdot e^{-0{,}3t}$

$$N = \frac{1}{0{,}0002 + c \cdot e^{-0{,}3t}} = \frac{5000}{1 + c_1 \cdot e^{-0{,}3t}}$$

$$N(0) = \frac{5000}{1 + c_1} = 4500$$

$4500 + 4500c_1 = 5000$

$4500c_1 = 500$

$c_1 = \frac{1}{9}$

$$N = \frac{5000}{1 + \frac{1}{9}e^{-0{,}3t}}$$

$N(10) = 4972$

b. de grenswaarde wordt nu dus 2500

$\frac{dN}{dt} = 0{,}00004N(2500 - N)$

$= 0{,}1N - 0{,}00004N^2$

stel $N = \frac{1}{u}$ dan $\frac{dN}{du} = -\frac{1}{u^2}$

$\frac{dN}{dt} = \frac{dN}{du} \cdot \frac{du}{dt} = -\frac{1}{u^2} \cdot \frac{du}{dt} = 0{,}1 \cdot \frac{1}{u} - 0{,}00004 \cdot \frac{1}{u^2}$

$\frac{du}{dt} = -0{,}1u + 0{,}00004$

$u = 0{,}0004$ is een particuliere oplossing

$\frac{du}{dt} = -0{,}1u$ geeft $u = c \cdot e^{-0{,}1t}$

$u = 0{,}0004 + c \cdot e^{-0{,}1t}$

$$N = \frac{1}{0{,}0004 + c \cdot e^{-0{,}1t}} = \frac{2500}{1 + c_1 \cdot e^{-0{,}1t}}$$

$$N(0) = \frac{2500}{1 + c_1 \cdot e^{-1}} = 4972$$

$4972 + 4972c_1 \cdot e^{-1} = 2500$

$4972c_1 \cdot e^{-1} = -2472$

$c_1 = 1{,}351$

$$N = \frac{2500}{1 - 1{,}351e^{-0{,}1t}}$$

$$\frac{2500}{1-1{,}351e^{-0{,}1t}}=4500$$

neem $y_1=\dfrac{2500}{1-1{,}351e^{-0{,}1x}}$ en $y_2=4500$

intersect geeft $x=11{,}1$

dus $11{,}1-10=1{,}1$ jaar na $t=10$ zijn er weer 4500 panda’s

15.5 Differentiaalvergelijkingen van de tweede orde

Opgave 60:

a. de tweede afgeleide is de hoogste afgeleide die in de differentiaalvergelijking voorkomt

b. $y(t) = e^{\lambda t}$

$y' = \lambda \cdot e^{\lambda t}$

$y'' = \lambda^2 \cdot e^{\lambda t}$

invullen in de dv geeft:

$\lambda^2 \cdot e^{\lambda t} + 5\lambda \cdot e^{\lambda t} + 4e^{\lambda t} = 0$

$(\lambda^2 + 5\lambda + 4) \cdot e^{\lambda t} = 0$

$\lambda^2 + 5\lambda + 4 = 0 \quad \vee \quad e^{\lambda t} = 0$ (k.n.)

dus $\lambda^2 + 5\lambda + 4 = 0$

$(\lambda + 4)(\lambda + 1) = 0$

$\lambda = -4 \quad \vee \quad \lambda = -1$

c. $y(t) = A \cdot e^{-t} + B \cdot e^{-4t}$

$y' = -A \cdot e^{-t} - 4B \cdot e^{-4t}$

$y'' = A \cdot e^{-t} + 16B \cdot e^{-4t}$

invullen in de dv geeft:

$A \cdot e^{-t} + 16B \cdot e^{-4t} - 5A \cdot e^{-t} - 20B \cdot e^{-4t} + 4A \cdot e^{-t} + B \cdot e^{-4t} = 0$

$0 = 0$ dus klopt

dus voor elke waarde van A en B voldoet $y(t) = A \cdot e^{-t} + B \cdot e^{-4t}$ aan de dv

Opgave 61:

a. $y'' + 4y' + 5y = 0$

$y = e^{\lambda t}$ dus $y' = \lambda \cdot e^{\lambda t}$ en $y'' = \lambda^2 \cdot e^{\lambda t}$

invullen in de dv geeft:

$\lambda^2 \cdot e^{\lambda t} + 4\lambda \cdot e^{\lambda t} + 5e^{\lambda t} = 0$

$(\lambda^2 + 4\lambda + 5) \cdot e^{\lambda t} = 0$

$\lambda^2 + 4\lambda + 5 = 0 \quad \vee \quad e^{\lambda t} = 0$ (k.n.)

$D = 4^2 - 4 \cdot 1 \cdot 5 = -4$ dus geen oplossingen

b. $\lambda^2 + 4\lambda + 5 = 0$

$(\lambda + 2)^2 - 4 + 5 = 0$

$(\lambda + 2)^2 = -1$

$\lambda + 2 = i \quad \vee \quad \lambda + 2 = -i$

$\lambda = -2 + i \quad \vee \quad \lambda = -2 - i$

Opgave 62:

λ_1 is oplossing van $\lambda^2 + p\lambda + q = 0$ dus geldt: ${\lambda_1}^2 + p\lambda_1 + q = 0$

λ_2 is ook een oplossing dus geldt: ${\lambda_2}^2 + p\lambda_2 + q = 0$

als $y = A \cdot e^{\lambda_1 t} + B \cdot e^{\lambda_2 t}$

dan $y' = \lambda_1 A \cdot e^{\lambda_1 t} + \lambda_2 B \cdot e^{\lambda_2 t}$

en $y'' = {\lambda_1}^2 A \cdot e^{\lambda_1 t} + {\lambda_2}^2 B \cdot e^{\lambda_2 t}$

invullen in de dv geeft:

$\lambda_1{}^2 A\cdot e^{\lambda_1 t}+\lambda_2{}^2 B\cdot e^{\lambda_2 t}+p(\lambda_1 A\cdot e^{\lambda_1 t}+\lambda_2 B\cdot e^{\lambda_2 t})+q(A\cdot e^{\lambda_1 t}+B\cdot e^{\lambda_2 t})=0$

$(\lambda_1{}^2+p\lambda_1+q)\cdot A\cdot e^{\lambda_1 t}+(\lambda_2{}^2+p\lambda_2+q)\cdot B\cdot e^{\lambda_2 t}=0$

$0=0$ klopt, dus alle functies van de vorm $y(t)=A\cdot e^{\lambda_1 t}+B\cdot e^{\lambda_2 t}$ zijn oplossing van de dv

Opgave 63:

λ_1 is oplossing van $\lambda^2+p\lambda+q=0$ dus geldt: $\lambda_1{}^2+p\lambda_1+q=0$ en omdat het de enige reële oplossing is geldt dat: $x_{top}=\frac{-p}{2}=-\frac{1}{2}p=\lambda_1$ dus $\lambda_1+\frac{1}{2}p=0$ ofwel $2\lambda_1+p=0$

stel $y=(A+Bt)\cdot e^{\lambda_1 t}$

dan $y'=B\cdot e^{\lambda_1 t}+(A+Bt)\cdot\lambda_1\cdot e^{\lambda_1 t}$

en $y''=B\cdot\lambda_1\cdot e^{\lambda_1 t}+B\cdot\lambda_1\cdot e^{\lambda_1 t}+(A+Bt)\cdot\lambda_1{}^2\cdot e^{\lambda_1 t}$

$=2B\cdot\lambda_1\cdot e^{\lambda_1 t}+(A+Bt)\cdot\lambda_1{}^2\cdot e^{\lambda_1 t}$

invullen in de dv geeft:

$2B\cdot\lambda_1\cdot e^{\lambda_1 t}+(A+Bt)\cdot\lambda_1{}^2\cdot e^{\lambda_1 t}+p(B\cdot e^{\lambda_1 t}+(A+Bt)\cdot\lambda_1\cdot e^{\lambda_1 t})+q(A+Bt)\cdot e^{\lambda_1 t}=0$

$(\lambda_1{}^2+p\lambda_1+q)(A+Bt)\cdot e^{\lambda_1 t}+B(2\lambda_1+p)\cdot e^{\lambda_1 t}=0$

$0\cdot(A+Bt)\cdot e^{\lambda_1 t}+B\cdot 0\cdot e^{\lambda_1 t}=0$

$0=0$ klopt, dus alle functies van de vorm $y=(A+Bt)\cdot e^{\lambda_1 t}$ zijn oplossing van de dv

Opgave 64:

$\lambda_1=a+bi$ is oplossing van $\lambda^2+p\lambda+q=0$ dus $(a+bi)^2+p(a+bi)+q=0$

$a^2+2abi-b^2+pa+pbi+q=0$

$a^2-b^2+pa+q+b(2a+p)i=0$

$a^2-b^2+pa+q=0 \quad\wedge\quad 2a+p=0$ want $b\neq 0$

$a^2-b^2+pa+q=0 \quad\wedge\quad p=-2a$

$a^2-b^2-2a^2+q=0 \quad\wedge\quad p=-2a$

$q=a^2+b^2 \quad\wedge\quad p=-2a$

$\lambda_2=a-bi$ is oplossing van $\lambda^2+p\lambda+q=0$ dus $(a-bi)^2+p(a-bi)+q=0$

$a^2-2abi-b^2+pa-pbi+q=0$

$a^2-b^2+pa+q-b(2a+p)i=0$

$a^2-b^2+pa+q=0 \quad\wedge\quad 2a+p=0$ want $b\neq 0$

$a^2-b^2+pa+q=0 \quad\wedge\quad p=-2a$

$a^2-b^2-2a^2+q=0 \quad\wedge\quad p=-2a$

$q=a^2+b^2 \quad\wedge\quad p=-2a$

stel $y(t)=(A\cos(bt)+B\sin(bt))\cdot e^{at}$

dus $y'=(-bA\sin(bt)+bB\cos(bt))\cdot e^{at}+(A\cos(bt)+B\sin(bt))\cdot ae^{at}$

$=(-bA\sin(bt)+bB\cos(bt)+aA\cos(bt)+aB\sin(bt))\cdot e^{at}$

dus $y''=(-b^2A\cos(bt)-b^2B\sin(bt)-abA\sin(bt)+abB\cos(bt)\cdot e^{at}+$

$(-bA\sin(bt)+bB\cos(bt)+aA\cos(bt)+aB\sin(bt))\cdot ae^{at}$

$=(a^2A\cos(bt)-b^2A\cos(bt)+a^2B\sin(bt)-b^2B\sin(bt)+2abB\cos(bt)-2abA\sin(bt))\cdot e^{at}$

invullen in de dv geeft:

$(a^2 A\cos(bt) - b^2 A\cos(bt) + a^2 B\sin(bt) - b^2 B\sin(bt) + 2abB\cos(bt) - 2abA\sin(bt)) \cdot e^{at} +$

$-2a(-bA\sin(bt) + bB\cos(bt) + aA\cos(bt) + aB\sin(bt)) \cdot e^{at} +$

$(a^2 + b^2)(A\cos(bt) + B\sin(bt)) \cdot e^{at} = 0$

haakjes wegwerken levert nu $0 = 0$ dus klopt

Opgave 65:

a. $y'' + 6y' + 8 = 0$

$\lambda^2 + 6\lambda + 8 = 0$

$(\lambda + 4)(\lambda + 2) = 0$

$\lambda = -4 \quad \vee \quad \lambda = -2$

dus $y = A \cdot e^{-4t} + B \cdot e^{-2t}$ en dus $y' = -4A \cdot e^{-4t} - 2B \cdot e^{-2t}$

$y(0) = 1$ dus $A + B = 1$

$y'(0) = -1$ dus $-4A - 2B = -1$

$$\begin{cases} A + B = 1 & \times 2 \\ -4A - 2B = -1 & \times 1 \end{cases}$$

$$\begin{cases} 2A + 2B = 2 \\ -4A - 2B = -1 \quad + \end{cases}$$

$-2A = 1$

$A = -\frac{1}{2}$

$B = 1\frac{1}{2}$

$y(t) = -\frac{1}{2}e^{-4t} + 1\frac{1}{2}e^{-2t}$

b. $y'' + 6y' + 10y = 0$

$\lambda^2 + 6\lambda + 10 = 0$

$(\lambda + 3)^2 - 9 + 10 = 0$

$(\lambda + 3)^2 = -1$

$\lambda + 3 = i \quad \vee \quad \lambda + 3 = -i$

$\lambda = -3 + i \quad \vee \quad \lambda = -3 - i$

dus $y(t) = (A\cos(t) + B\sin(t)) \cdot e^{-3t}$

dus $y' = (-A\sin(t) + B\cos(t)) \cdot e^{-3t} + (A\cos(t)) + B\sin(t)) \cdot -3e^{-3t}$

$= ((B - 3A)\cos(t) + (-A - 3B)\sin(t)) \cdot e^{-3t}$

$y(0) = 1$ geeft: $A = 1$

$y'(0) = -1$ geeft: $B - 3A = -1$

$B = 3A - 1 = 3 - 1 = 2$

dus $y(t) = (\cos(t) + 2\sin(t)) \cdot e^{-3t}$

c. $y'' - 8y' + 16 = 0$

$\lambda^2 - 8\lambda + 16 = 0$

$(\lambda - 4)^2 = 0$

$\lambda = 4$

dus $y(t) = (A + Bt) \cdot e^{4t}$

dus $y'(t) = B \cdot e^{4t} + (A + Bt) \cdot 4e^{4t}$

$y(0) = 1$ dus $A = 1$

$y'(0) = -1$ dus $B + 4A = -1$

$B = -4A - 1 = -4 - 1 = -5$

dus $y(t) = (1 - 5t) \cdot e^{4t}$

d. $y'' + \frac{1}{4} y = 0$

$\lambda^2 + \frac{1}{4} = 0$

$\lambda^2 = -\frac{1}{4}$

$\lambda = \frac{1}{2} i \quad \vee \quad \lambda = -\frac{1}{2} i$

$y(t) = (A\cos(\frac{1}{2}t) + B\sin(\frac{1}{2}t)) \cdot e^{0t} = A\cos(\frac{1}{2}t) + B\sin(\frac{1}{2}t)$

dus $y'(t) = -\frac{1}{2} A\sin(\frac{1}{2}t) + \frac{1}{2} B\cos(\frac{1}{2}t)$

$y(0) = 1$ geeft: $A = 1$

$y'(0) = -1$ geeft: $\frac{1}{2} B = -1$

dus $B = -2$

dus $y(t) = \cos(\frac{1}{2}t) - 2\sin(\frac{1}{2}t)$

Opgave 66:

a. $x = -\frac{dy}{dt} + y$

differentiëren geeft: $\frac{dx}{dt} = -\frac{d^2y}{dt^2} + \frac{dy}{dt}$

invullen in $\frac{dx}{dt} = 2x - 6y$ geeft:

$-\frac{d^2y}{dt^2} + \frac{dy}{dt} = 2\left(-\frac{dy}{dt} + y\right) - 6y$

$-\frac{d^2y}{dt^2} + \frac{dy}{dt} = -2\frac{dy}{dt} + 2y - 6y$

$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} - 4y = 0$

b. $y'(0) = \left[\frac{dy}{dt}\right]_{t=0} = -x(0) + y(0) = -1 - 1 = -2$

c. $y'' - 3y' - 4y = 0$

$\lambda^2 - 3\lambda - 4 = 0$

$(\lambda - 4)(\lambda + 1) = 0$

$\lambda = 4 \quad \vee \quad \lambda = -1$

$y = A \cdot e^{4t} + B \cdot e^{-t}$ dus $y' = 4A \cdot e^{4t} - B \cdot e^{-t}$

$y(0) = A + B = -1 \qquad y'(0) = 4A - B = -2$

$$\begin{cases} A + B = 1 \\ 4A - B = -2 \quad + \end{cases}$$

$5A = -1$

$A = -\frac{1}{5}$ dus $B = 1\frac{1}{5}$

$y(t) = -\frac{1}{5} e^{4t} + 1\frac{1}{5} e^{-t}$

$y' = -\frac{4}{5} e^{4t} - 1\frac{1}{5} e^{-t}$

$x = -\frac{dy}{dt} + y$

$= -\left(-\frac{4}{5} e^{4t} - 1\frac{1}{5} e^{-t}\right) + -\frac{1}{5} e^{4t} + 1\frac{1}{5} e^{-t}$

$= \frac{4}{5} e^{4t} + 1\frac{1}{5} e^{-t} - \frac{1}{5} e^{4t} + 1\frac{1}{5} e^{-t}$

$= \frac{3}{5} e^{4t} + 2\frac{2}{5} e^{-t}$

Opgave 67:

a. als je het blokje loslaat dan gaat het naar boven, dus F is naar boven gericht en dus positief, omdat $u < 0$ is $c > 0$

b. $F = m \cdot a = m \cdot \frac{dv}{dt}$

$\frac{dv}{dt} = \frac{F}{m} = \frac{-c \cdot u}{m}$

c. $\frac{dv}{dt} = \frac{-1 \cdot u}{1} = -u$

v is de snelheid waarmee u verandert, dus $\frac{du}{dt} = v$

het blokje wordt naar beneden getrokken tot $u = -1$ en wordt op $t = 0$ losgelaten

dus $u(0) = -1$ en $v(0) = 0$

d. $\frac{du}{dt} = v$ dus $\frac{d^2u}{dt^2} = \frac{dv}{dt}$

$\frac{dv}{dt} = -u$ dus $\frac{d^2u}{dt^2} = -u$

ofwel $\frac{d^2u}{dt^2} + u = 0$

$\lambda^2 + 1 = 0$

$\lambda^2 = -1$

$\lambda = i \quad \vee \quad \lambda = -i$

dus $u(t) = (A\cos t + B\sin t) \cdot e^{0t} = A\cos t + B\sin t$

$u(0) = A = -1$

$u'(t) = -A\sin t + B\cos t$

$u'(0) = B = 0$

dus $u(t) = -\cos t$

Opgave 68:

a. $F = -40\frac{du}{dt} - 72u$ en $\frac{d^2u}{dt^2} = \frac{F}{m} = \frac{F}{2} = \frac{1}{2}F$

$\frac{d^2u}{dt^2} = -20\frac{du}{dt} - 36u$

$\frac{d^2u}{dt^2} + 20\frac{du}{dt} + 36u = 0$

$\lambda^2 + 20\lambda + 36 = 0$

$(\lambda + 2)(\lambda + 18) = 0$

$\lambda = -2 \quad \vee \quad \lambda = -18$

$u(t) = A \cdot e^{-2t} + B \cdot e^{-18t}$

$u(0) = A + B = 1$

$u'(t) = -2A \cdot e^{-2t} - 18B \cdot e^{-18t}$

$v(0) = u'(0) = -2A - 18B = 2$

$$\begin{cases} A + B = 1 & \times 2 \\ -2A - 18B = 2 & \times 1 \end{cases}$$

$$\begin{cases} 2A + 2B = 2 \\ -2A - 18B = 2 \quad + \end{cases}$$

$-16B = 4$

$B = -\frac{1}{4}$ dus $A = 1\frac{1}{4}$

$u(t) = 1\frac{1}{4}e^{-2t} - \frac{1}{4}e^{-18t}$

b. $D = \dfrac{k^2 - 4cm}{m^2} = \dfrac{40^2 - 4\cdot 72\cdot 8}{8^2} = -11 < 0$ dus onderdemping ofwel een gedempte trilling

c. $D \geq 0$

$D = \dfrac{k^2 - 4cm}{m^2} = \dfrac{1600 - 288m}{m^2} \geq 0$

de noemer is altijd positief, dus moet gelden:

$1600 - 288m \geq 0$

$-288m \geq -1600$

$m \leq 5\frac{5}{9}$

Opgave 69:

a. $F = -1000\frac{du}{dt} - 500u$ en $\frac{d^2u}{dt^2} = \dfrac{F}{m} = \dfrac{F}{1000}$

$\frac{d^2u}{dt^2} = -\frac{du}{dt} - \frac{1}{2}u$

$\frac{d^2u}{dt^2} + \frac{du}{dt} + \frac{1}{2}u = 0$

$\lambda^2 + \lambda + \frac{1}{2} = 0$

$(\lambda + \frac{1}{2})^2 - \frac{1}{4} + \frac{1}{2} = 0$

$(\lambda + \frac{1}{2})^2 = -\frac{1}{4}$

$\lambda + \frac{1}{2} = \frac{1}{2}i \quad \vee \quad \lambda + \frac{1}{2} = -\frac{1}{2}i$

$\lambda = -\frac{1}{2} + \frac{1}{2}i \quad \vee \quad \lambda = -\frac{1}{2} - \frac{1}{2}i$

$u(t) = (A\cos(\frac{1}{2}t) + B\sin(\frac{1}{2}t)) \cdot e^{-\frac{1}{2}t}$

$u(0) = A = 1$

$u'(t) = (-\frac{1}{2}A\sin(\frac{1}{2}t) + \frac{1}{2}B\cos(\frac{1}{2}t)) \cdot e^{-\frac{1}{2}t} + (A\cos(\frac{1}{2}t) + B\sin(\frac{1}{2}t)) \cdot -\frac{1}{2}e^{-\frac{1}{2}t}$

$u'(0) = \frac{1}{2}B - \frac{1}{2}A = 0$

$\frac{1}{2}B = \frac{1}{2}A$

$B = A = 1$

$u(t) = (\cos(\frac{1}{2}t) + \sin(\frac{1}{2}t)) \cdot e^{-\frac{1}{2}t}$

b. $D \geq 0$

$D = \dfrac{k^2 - 4cm}{m^2} = \dfrac{k^2 - 4\cdot 500\cdot 1000}{1000^2} = \dfrac{k^2 - 2000000}{1000000} \geq 0$

$k^2 - 2000000 \geq 0$

$k^2 \geq 2000000$

$k \leq -\sqrt{2000000} = -1000\sqrt{2} \quad \vee \quad k \geq 1000\sqrt{2}$

dus $k \geq 1000\sqrt{2}$

Opgave 70:

a. $F = 0\cdot\frac{du}{dt} - c\cdot u$

$\frac{d^2u}{dt^2} = \dfrac{F}{m} = -\dfrac{c}{m}\cdot u$

$$\frac{d^2u}{dt^2} + \frac{c}{m} \cdot u = 0$$

$$\lambda^2 + \frac{c}{m} = 0$$

$$\lambda^2 = -\frac{c}{m}$$

$$\lambda^2 = \frac{c}{m} \cdot i^2$$

$$\lambda = i \cdot \sqrt{\tfrac{c}{m}} \quad \vee \quad \lambda = -i \cdot \sqrt{\tfrac{c}{m}}$$

$$u(t) = A\cos\left(t \cdot \sqrt{\tfrac{c}{m}}\right) + B\sin(t \cdot \sqrt{\tfrac{c}{m}})$$

$$periode = \frac{2\pi}{\sqrt{\tfrac{c}{m}}} = 2\pi \cdot \sqrt{\tfrac{m}{c}}$$

dus $T = 2\pi \cdot \sqrt{\tfrac{m}{c}}$

b. $F = -k \cdot \frac{du}{dt} - c \cdot u$

$$\frac{d^2u}{dt^2} = \frac{F}{m} = -\tfrac{k}{m} \cdot \tfrac{du}{dt} - \tfrac{c}{m} \cdot u$$

$$\frac{d^2u}{dt^2} + \tfrac{k}{m} \cdot \tfrac{du}{dt} + \tfrac{c}{m} \cdot u = 0$$

$$\lambda^2 + \tfrac{k}{m} \cdot \lambda + \tfrac{c}{m} = 0$$

$D \geq 0$ geeft $\left(\tfrac{k}{m}\right)^2 - 4 \cdot \tfrac{c}{m} \geq 0$

$$(\tfrac{k}{m})^2 - 4c \cdot m \geq 0$$

$$(\tfrac{k}{m})^2 \geq 4c \cdot m$$

$$\tfrac{k}{m} \geq 2\sqrt{c \cdot m}$$

Opgave 71:

a. $D = \dfrac{k^2 - 4cm}{m^2} = 0$

$$k^2 - 4cm = 0$$

$$k^2 = 4cm$$

$$k = 2\sqrt{cm} = 2\sqrt{25000 \cdot 30} = 1732$$

b. $F = -1730 \cdot \frac{du}{dt} - 25000u$

$$\frac{d^2u}{dt^2} = \frac{F}{50}$$

$$\frac{d^2u}{dt^2} = -34{,}6\frac{du}{dt} - 500u$$

$$\frac{d^2u}{dt^2} + 34{,}6\frac{du}{dt} + 500u = 0$$

$$\lambda^2 + 34{,}6\lambda + 500 = 0$$

$$(\lambda + 17{,}3)^2 - 299{,}29 + 500 = 0$$

$$(\lambda + 17{,}3)^2 = -200{,}71$$

$$\lambda + 17{,}3 = i\sqrt{200{,}71} \quad \vee \quad \lambda + 17{,}3 = -i\sqrt{200{,}71}$$

$$\lambda = -17{,}3 + i\sqrt{200{,}71} \quad \vee \quad \lambda = -17{,}3 - i\sqrt{200{,}71}$$

$\lambda = -17{,}3 + 14{,}2i \quad \vee \quad \lambda = -17{,}3 - 14{,}2i$

$u = (A\cos(14{,}2t) + B\sin(14{,}2t)) \cdot e^{-17{,}3t}$

dus de eigenfrequentie is: $\frac{14{,}2}{2\pi} = 2{,}3$ Hz

c. 70 omwentelingen per minuut is 1,17 omwentelingen per seconde
het veersysteem krijgt dus ongeveer 2,34 keer per seconde een duw naar beneden, waardoor resonantie kan ontstaan.

Opgave 72:

a. $D = \dfrac{k^2 - 4cm}{m^2} = \dfrac{k^2 - 4 \cdot 200000 \cdot 1000}{1000^2} = \dfrac{k^2 - 800000000}{1000000} = 0$

$k^2 - 800000000 = 0$

$k^2 = 800000000$

$k = \sqrt{800000000} \quad \vee \quad k = -\sqrt{800000000}$ (k.n.)

$k = 20000\sqrt{2}$

b. $F = -20000\sqrt{2} \cdot \frac{du}{dt} - 200000u$

$\frac{d^2u}{dt^2} = \dfrac{F}{1000}$

$\frac{d^2u}{dt^2} = -20\sqrt{2}\,\frac{du}{dt} - 200u$

$\frac{d^2u}{dt^2} + 20\sqrt{2}\,\frac{du}{dt} + 200u = 0$

$\lambda^2 + 20\sqrt{2}\lambda + 200 = 0$

$\lambda = \dfrac{-20\sqrt{2} \pm \sqrt{0}}{2} = -10\sqrt{2}$

$u(t) = (A + Bt) \cdot e^{-10\sqrt{2}t}$

$u(0) = A = 1{,}5$

$u' = B \cdot e^{-10\sqrt{2}t} + (A + Bt) \cdot e^{-10\sqrt{2}t} \cdot -10\sqrt{2}$

$u'(0) = B - 10\sqrt{2} \cdot A = 0$

$B = 10\sqrt{2} \cdot A = 10\sqrt{2} \cdot 1{,}5 = 15\sqrt{2}$

$u(t) = (1{,}5 + 15\sqrt{2} \cdot t) \cdot e^{-10\sqrt{2}t}$

c. $u(t) = (1{,}5 + 15\sqrt{2} \cdot t) \cdot e^{-10\sqrt{2}t} = 0{,}01$

neem $y_1 = (1{,}5 + 15\sqrt{2} \cdot x) \cdot e^{-10\sqrt{2}x}$ en $y_2 = 0{,}01$

intersect geeft $x = 0{,}502$

dus na ongeveer 0,5 seconde is de uitwijking minder dan 1 cm